Using Computer Aided Geometric Design tools in dynamical modelling of self gravitating systems.

F. I. Diakogiannis, G. F. Lewis, R. A. Ibata
Introducing CAGD tools: B-spline curves

B-spline curves and functions: Polynomial pieces joined together.

\[ \frac{dP(x)}{dx} = \sum_{n} \frac{d}{dx} B_{i,n}(x) \]

where

- \( P(x) \) is the curve function.
- \( B_{i,n}(x) \) are the B-spline basis functions.
- \( x \) is the parameter along the curve.
- \( i \) and \( n \) are indices.

The diagram illustrates the construction of a B-spline curve using control points and the concept of blending functions. The curve is generated by a weighted sum of these basis functions, where the weights are determined by the control points.
Modelling with smoothing splines: The statistician's way (nonparametric regression)

Example: 500 data points

\[ y_j = f_{\text{ref}}(x_j) + e_j \]

Method:

\[ f_{\text{ref}}(x) = e^{-\frac{x}{20\pi}} \cos\left(\frac{x}{2\pi}\right) \]
Physical System: Depends on some variable $x$ and some parameter $\theta$

Comparison of $\text{observable}(x|\theta)$ function with data, gives us distribution of parameter $\theta$
CAD tools in dynamical modelling: Key Idea

Problem:
• Wrong assumption of functional form of \( \text{have_no_idea}(x|\theta) \) gives significantly biased estimates of parameter \( \theta \).
• We have no data to compare directly to \( \text{have_no_idea}(x|\theta) \)

Solution:
• \( \text{have_no_idea}(x|\theta) \) curve is **not arbitrary**: it is convolved with the \( \text{good_guess}(x|\theta) \) through some very complicated equations. This is **VERY** important information for a physical system and a severe restriction to \( \text{have_no_idea}(x|\theta) \) function.
• Approximate \( \text{have_no_idea}(x|\theta) \) with a smooth CAD curve and allow information from data (statistics) and knowledge of the physical system (**FUNDAMENTAL** equations of physics) to both define the shape and estimate the values.
• Fundamental equations of physics (e.g. conserved quantities) introduce less bias in key parameters (e.g. mass of a stellar cluster).
• Define “smoothing penalty” from information of ideal theoretical models.
• Explore parameter space with MCMC in order to see if there is inherent degeneracy to the problem (multiple solutions).
Connecting observables with theory

Velocity dispersion $\sigma_{\text{los}}^2(R)$ + Spatial distribution of Stars \[ \left( \frac{\Sigma(R_j)}{\Upsilon} \right) \]
Connecting observables with theory

Velocity dispersion $\sigma_{\text{los}}^2(R)$ + Spatial distribution of Stars

Marginalized distributions of model defining parameters $\theta$
The mass anisotropy degeneracy of the SSJE

Spherically Symmetric Jeans Equation:

\[- \frac{d\Phi}{dr} = \frac{1}{\rho} \frac{d(\rho \sigma_{rr}^2)}{dr} + \frac{2\beta(r)}{r} \sigma_{rr}^2\]

where \( \beta = 1 - \frac{\sigma_{tt}^2}{2\sigma_{rr}^2} \)

Connection with observables:

\[\sigma_{los}^2(R) = \frac{2}{\Sigma(R)} \int_R^{r_t} \left( 1 - \beta(r) \frac{R^2}{r^2} \right) \frac{r \rho \sigma_{rr}^2}{\sqrt{r^2 - R^2}} dr\]

Diakogiannis, Lewis, Ibata (2014a,b), subm. MNRAS.
Manipulating the Spherically Symmetric Jeans equation

\[ \sigma_{los}^2(R) = \sum_i a_i I_i(R) + C(R) \]

\[ \sigma_{los}^2 = \frac{1}{\Sigma(R)} \int_R^{r_t} \rho(r) R^2 \frac{d\Phi(r)}{dr} \]

\[ C(R) = \frac{1}{\Sigma(R)} \int_R^{r_t} \frac{\rho(r) R^2}{\sqrt{r^2 - R^2}} \frac{d\Phi(r)}{dr} \]

\[ I_i(R) = \frac{1}{\Sigma(R)} \int_R^{r_t} \frac{\left(2r \rho + \rho^{(1)} R^2\right) B_{i,k}(r) + \rho R^2 B_{i,k}^{(1)}(r)}{\sqrt{r^2 - R^2}} \]

\[ -\frac{d\Phi}{dr} = \frac{d\psi}{dr} + \frac{1}{\rho} \frac{d\rho}{dr} \psi + \frac{2}{r} \psi - \frac{1}{r} \phi \]

\[ \psi(r) = \sum_{i=1}^{N_{\text{coeffs}}} a_i B_{i,k}(r) \]

\[ \frac{d\psi}{dr} = \sum_{i=1}^{N_{\text{coeffs}}} a_i \frac{dB_{i,k}(r)}{dr} \]
Application to synthetic data: Single component system const mass-to-light ratio $Y$, smoothing penalty

**D. Ossipkov Merritt β** – King mass model, 14 data points

Mass content fully reconstructed
Application to synthetic data: Stellar+DM system, smoothing penalty

D. Ossipkov Merritt $\beta$ – King+NFW mass model, 14 data points

Mass content fully reconstructed
Work in progress...

\[
[\sigma_{los}^2(R^\alpha)] = \left(\left[\begin{array}{c} X_1^\alpha \\ \vdots \\ X_2^\alpha \end{array}\right] \right) \cdot \left(\begin{array}{c} c_j^* \\ b_k^* \end{array}\right) + [D^\alpha]
\]
First fit $f(E,x)$ DF, from fitting mass density $\rho(r)$

- true
- FIT
Discussion

- When we don't know the functional form and we don't have data to make an educated guess → Combine CAD with fundamental equations of physics.
- Smaller bias: results from freedom of flexibility and generality of fundamental eq.
- Explore parameter space with MCMC → Unique/Multimodal solutions.
- Freedom of flexibility may reveal regions of interest of physical systems → can lead to new discoveries. Remember: NO data to compare to have_no_id!!!
- Use information from IDEAL theoretical models for optimum smoothing.

Difficulties (the joy of computing):
- Needs combination of statistics and optimization (Genetic Algorithms) to find optimum control points placement. This reduces dimensionality of the problem and gives optimum results.

THANK YOU! :)

For self gravitating stellar systems:
- It will allow for (more...) accurate DM estimates of dSph and Gcs
- It will allow for the CORRECT shape of the DM mass profile!!
- It can help us reconstruct the full DF of a system from observables.. - (Sch/IId)
- Computationally efficient