Electrodynamics of solar flares and pulsars

What errors do plasma astrophysicists introduce by using only half of Maxwell’s equations?

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The problem:

- Models for solar flares are time-independent
  - $\Rightarrow$ they omit the inductive electric field, $E_{\text{ind}}$
- Models for pulsar magnetospheres also ignore $E_{\text{ind}}$
- Does it matter that we use wrong electrodynamics?
  - Patch-work models are misleading
  - Not useful as predictive tools, e.g., for flare stars, giant bursts on SGRs, superflares & FRBs

Must include both $E_{\text{ind}}$ & plasma response to it

Locally:

- $E_{\text{ind}}$ drives drift $u_{\text{ind}} = E_{\text{ind}} \times B / B^2$
- $\partial E_{\text{ind}} / \partial t$ drives a polarization current
  $$J_{p\text{ol}} = \left( \frac{c^2}{v_A^2} \right) \varepsilon_0 \partial E_{\text{ind}} / \partial t$$

Globally:

- Integrated forms of Maxwell’s equations
- EMF drives current around some circuit
  - $\Rightarrow$ circuit concepts cannot be avoided
Changing magnetic fields

Solar flares (and other magnetic explosions) and pulsars have strong magnetic fields changing rapidly in time ⇒ a large inductive electric field that we ignore

Models for flares are time-stationary
So are models for pulsars when the plasma is included

When time-dependence is omitted
electrodynamics reduces to electrostatics
⇒ inductive electric field is ignored

How big is the effect we are ignoring?

Integrated form of Faraday's equation: \( \text{EMF } \Phi = -d\Psi_{\text{mag}}/dt \)
Solar flares, \( \Phi = 10^9-10^{10} \text{ V} \); pulsars, \( \Phi = 10^{12}-10^{16} \text{ V} \)
Maxwell’s equations

The terms in red are neglected in astrophysical electrodynamics when both a large-scale $\mathbf{B}$ & a plasma are included:

\[
\begin{align*}
\text{curl } \mathbf{E}_{\text{ind}} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\text{curl } \mathbf{B} &= \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}_{\text{ind}}}{\partial t} \\
\text{div } \mathbf{B} &= 0 \\
\text{div } \mathbf{E}_{\text{pot}} &= \frac{\rho}{\varepsilon_0}
\end{align*}
\]

Neglected are:
- the inductive electric field $\mathbf{E}_{\text{ind}}$
- & the displacement current $\mathbf{J}_{\text{disp}} = \varepsilon_0 \frac{\partial \mathbf{E}_{\text{ind}}}{\partial t}$

Correct electrodynamics requires inductive & potential fields

In the Coulomb gauge $\mathbf{E} = \mathbf{E}_{\text{ind}} + \mathbf{E}_{\text{pot}},$

\[
\begin{align*}
\mathbf{E}_{\text{ind}} &= -\frac{\partial \mathbf{A}}{\partial t}, \\
\mathbf{E}_{\text{pot}} &= -\text{grad } \phi.
\end{align*}
\]

Integrating $\mathbf{E}$ around closed loop
- only $\mathbf{E}_{\text{ind}}$ contributes to $\Phi$
Solar flares

What does a model need to explain?

- Magnetic energy builds up slowly (in active region)
- Released explosively in a flare
  - coronal mass ejection (CME) in eruptive flares
  - nonthermal component in all flares
- Nonthermal energy in electrons
  \[ \varepsilon = 10 \text{ keV at } \dot{N} = 10^{36} \text{ s}^{-1} \implies \varepsilon \dot{N} = 10^{21} \text{ W} \]

“Number problem”

- \( \Phi = 10^{10} \text{ V}, I = 10^{11} \text{ A} \implies I \Phi = 10^{21} \text{ W} \)
  \[ \varepsilon = \frac{e \Phi}{M}, e \dot{N} = MI, M = 10^6 \]
- Simple models wrong by six orders of magnitude!

Three classes of model (all time-independent)

- CSHKP models
- circuit models
- quadrupolar models
Figure: Top: a CSHKP model. Bottom: Petschek reconnection model.
Fig. 5. Illustrates an equivalent circuit for a closed loop. $V_G$ and $R_G$ represent the current generator voltage and resistive load, respectively. $R_f$ the resistive load in the other foot of the loop, $C$ the capacitance associated with the polarizability of a magnetized plasma, and $L$ the total inductance of a loop.

**Figure:** A circuit model for a flare (Spicer 1982)
Figure: A quadrupolar flare model (Nishio et al. 1997)
Solar-type model for giant burst on SGR

Figure: Cartoon for giant burst on a SGR (Duncan & Thompson 1995)

Analogous to solar circuit/photospheric dynamo model
Superflares on solar-type stars

Figure: Kepler data: white light flares, energy $10^{-7}$ times solar

Need a scaled solar flare model to apply to superflares
Pulsar electrodynamics

Over 2000 radio pulsars with known $P$, $\dot{P}$

Two incompatible models used for different purposes

Vacuum dipole model (VDM)

▶ Used to estimate $B \propto (P\dot{P})^{1/2}$, characteristic age $\propto P/\dot{P}$, pulsar potential $\Phi \propto (\dot{P}/P^3)^{1/2}$
▶ EM field can be calculated exactly

Rotating magnetosphere model (RMM)

▶ Aligned: magnetic and rotation axes assumed the same
▶ Plasma assumed to corotate with star $u_{\text{cor}} = \omega \times x$
▶ Frozen-in assumption $\Rightarrow E_{\text{cor}} = -(\omega \times x) \times B$
▶ ‘Charge starvation’: cannot provide $\rho_{\text{GJ}} = \varepsilon_0 \text{div } E_{\text{cor}}$
▶ ‘Gap’ forms; charges accelerated; pair cascade triggered
▶ Magnetosphere populated by secondary pairs

Both VDM & RMM fatally flawed as stand-alone models
Vacuum dipole model (VDM)

Rotating magnetic dipole in vacuo: fields known exactly

\[ n = \frac{x}{r}, \quad \dot{m} = \omega \times m, \quad \ddot{m} = \omega \times (\omega \times m) \]

\[ B(t, x) = \frac{\mu_0}{4\pi} \left[ \frac{3n \cdot m - m}{r^3} + \frac{3n \cdot \dot{m} - \ddot{m}}{r^2c} + \frac{n \times (n \times \dddot{m})}{rc^2} \right] \]

\[ E(t, x) = \frac{\mu_0}{4\pi} \left[ \frac{n \times \dot{m}}{r^2} + \frac{n \times \dddot{m}}{rc} \right] \]

Inductively induced drift velocity

\[ E_{\text{ind}}(t, x) = \frac{\mu_0 n \times (\omega \times m)}{4\pi r^2} \]

\[ E_{\text{ind}} \text{ cannot be screened by charges} \]

\[ => \text{electric drift } u_{\text{ind}} = E_{\text{ind}} \times B / B^2 \]

\[ |u_{\text{ind}}| \ & |u_{\text{cor}}| \text{ comparable in magnitude qualitatively different} \]

\[ u_{\text{ind}} \text{ unjustifiably neglected in existing models} \]
Non-corotating model synthesizes VDM & RMM

Plasma velocity includes arbitrary factor \(0 \leq y \leq 1\)

\[
u = (1 - y)u_{\text{cor}} + yu_{\text{ind}}, \quad u_{\text{cor}} = \frac{E_{\text{cor}} \times B}{B^2}, \quad u_{\text{ind}} = \frac{E_{\text{ind}} \times B}{B^2}
\]

Abrupt changes in \(y\) may explain abrupt changes in \(\dot{P}\)

**Figure:** \(\dot{P}\) changes when PSR B1931+24 nulls (Kramer et al. 2006)
Figure: Phase of drifting subpulses remembered across a null
Conclusions

Using only half of Maxwell’s equations is not justified in general.

Time-dependent $\mathbf{B} \Rightarrow \mathbf{E}_{\text{ind}} \neq 0$

Implications of neglect of $\mathbf{E}_{\text{ind}}$ & $\mathbf{J}_{\text{disp}}$:

1. Ignores the EMF $\Phi$ that drives a flare

Existing flare models not useful as analogs for magnetic explosions

2. Pulsar plasma cannot be corotating

Directly relevant to subpulse drifting & abrupt changes in $\dot{P}$

Time-dependent EM theory must be used to model astrophysical plasmas with time-dependent large-scale magnetic fields

Our models must be based on correct physics
Figure 3. Idealized quadrupolar model for a solar flare: (a) initial loops are in gray, final loops are in black; (b) projection onto the solar surface with the initial loops shown as thick solid lines, the final loops as dashed lines, and reconnected flux loops moving from one to the other, shown by thin lines. The EMF is assumed to drive current around the two triangles in (b), requiring current closure across field lines at each of the four footpoints (after Melrose 1997).
Acceleration by parallel electric field

Simplest acceleration mechanism for energetic particles:

electric field, $E_\parallel$, parallel to magnetic field.

Compelling evidence for this mechanism for auroral electrons,
accelerated 1–3 km above the Earth’s surface.

Strong evidence for $E_\parallel$-acceleration of electrons in solar flares.

**Problem:** acceleration by a potential fields does not work.

“What goes up must come down.”

$\Rightarrow$ Energy gained is lost on escaping acceleration region.

Does not apply to acceleration by inductive electric field.
Quote from Song & Lysak, PRL 96, 145002 (2006), in the context of auroral acceleration:

“... appeal to generalized Ohm’s law for $E_\parallel$ generation has misled and hindered research on reconnection and auroral acceleration ... processes causing and supporting $E_\parallel$ ... either neglected or not yet discovered (Fälthammar 1990)”

“$E_\parallel$ acceleration is associated with ... the parallel displacement current”

We still do not understand acceleration by $E_\parallel$.

The problem lies with the use of MHD.

A future successful theory must involve $E_{\text{ind}}$. 
Compelling observational evidence and theoretical arguments that pulsars magnetospheres do not corotate. But we have no plausible alternative assumption to corotation.

**Electrodynamics does not require corotation**

- Rotating magnetic field satisfies
  \[
  \frac{\partial \mathbf{B}(t, \mathbf{x})}{\partial t} = \text{curl} [(\mathbf{\omega} \times \mathbf{x}) \times \mathbf{B}(t, \mathbf{x})] = -\text{curl} \mathbf{E}
  \]

- Solution includes arbitrary potential field (Mestel 1971)
  \[
  \mathbf{E} = \mathbf{E}_{\text{cor}} - \text{grad} \, \psi = \mathbf{E}_{\text{ind}} - \text{grad} \, \phi.
  \]

- Fluid velocity \( \mathbf{u} = \mathbf{E} \times \mathbf{B} / B^2 \) not uniquely determined